New issues for Numerical Stochastic Perturbation Theory

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First attempts in the application of Numerical Stochastic Perturbation Theory (NSPT) to the problem of pushing one loop further the computation of SU(3) (SU(2)) pertubative β function (in different schemes) are reviewed and the relevance of such a computation is discussed. Other issues include the proposal of a different strategy for gauge-fixed NSPT computations in lattice QCD.

1. Perturbative β-functions by NSPT.

Perturbative calculations in lattice field theories are necessary to connect results in different renormalisation schemes. In asymptotically free theories particulary useful are matchings to those schemes in which perturbative computations at high energies are more easily achieved (first of all $\overline{\rm MS}$). It is well known that the bare definition of the coupling in lattice QCD is often not the best one: other definitions are useful, for instance, to relate the perturbative and the non perturbative regions in a smoother way.

In the case of QCD with no (or massless) quarks, or in any other pure gauge theory, just one free parameter is present and every coupling constant (in any scheme) can be expanded as a power series in any other. The first coefficient of such an expansion determines the relation between the physical scales (Λ parameter), the others are related to combinations of the respective (non universal) coefficients of the β-function.

Aim of the project is to obtain higher order expansions of convenient couplings in the lattice scheme, thus obtaining informations on higher order β-function coefficients. The natural goal is β_3 in the lattice scheme, which would result for example in a more precise correction to asymptotic scaling and in pushing one loop further the matching to $\overline{\rm MS}$.

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2. NSPT for lattice QCD

Unfortunately perturbation theory presents many difficulties in the lattice regularisation. To bypass such difficulties NSPT was introduced, which proved successful in performing high loops gauge invariant computations in lattice QCD [1].

The idea which makes NSPT possible is the following: The Langevin equation provides a quantisation scheme for field theories, and also a tool for numerical computations of the Functional Integral. It may be introduced by giving the fields a dependence on a new parameter (stochastic time), $U(\mu,x) \longrightarrow U_\eta(\mu,x,t)$, $x \in R^4$ ($\eta(t)$ is an appropriate random process), and then imposing the fields to obey the Langevin equation,

$$\frac{\partial}{\partial t}U_\eta = (-i\nabla S[U_\eta]_{|_{\mu,x,t}} - i\,\eta)U_\eta \qquad (1)$$

which determines the evolution of the fields in stochastic time. One then notices in the Langevin equation the explicit dependence on g through the action S; as a result the solution U_η depends on g as well and can be formally expanded as a power series in g. This produces a cascade of stochastic equations, of which only the first depends explicitily on the noise η and each one depends only on the lower order fields.

3. Symanzik's analysis

In order to get a continuum–limit result from perturbative calculations on the lattice it is nec-

essary to recognize in the data the dependence on the cutoff. Typically one computes a coupling which is defined at a length scale L on a lattice of size $I = L/a$, thus obtaining a generic perturbative coefficient $m(I)$; then Symanzik's analysis suggests an asymptotic expansion for $m(I)$ in the form: $m(I) = \sum_{n=0}^{\infty} \sum_{k=0}^{l} \frac{c_{n,k} \log(I)^k}{I^n}$, where the maximum power of the log is fixed by the order of the computation. It is actually this analysis that sets the required precision of the computation.

4. NSPT for the Schroedinger Functional scheme

One particulary interesting renormalisation scheme is introduced via the running coupling $\bar{g}^2(L)$ introduced in [2] and based on the Schroedinger Functional with an induced background field, obtained by fixing boundary conditions in the $t = 0$ and $t = L$ time–slices. The coupling $\bar{g}^2(L)$ is defined through the rensponse to a change in the boundary conditions [2]. Through a finite size scaling technique [3] the ALPHA Collaboration has been able to compute this coupling on a huge momentum range up to the perturbative domain in which contact can be made with other schemes such as $\overline{\rm MS}$. Beeing this scheme a successful one with many respects and having perturbative computations of this coupling already been performed at two loop level (so that crosschecks are possible), it was a natural candidate for our analysis.

It is however the first time that NSPT is applied starting from a vacuum which is not the trivial one. In fact the fields have to be represented as perturbative expansion around the classical solution induced by the background field $U_\eta = V + \sum_{k>0} \beta^{-\frac{k}{2}} U_\eta^{(k)}$. This poses no problem in principle, but it will be interesting to see which kind of effects such a modification brings to the precision of the results. As far as the observable is concerned, it is known to exhibit quite strong fluctuations even at non–perturbative level. A feature that immediately emerges is that it is effectively defined only on the two boundary time slices, and not on the bulk.

As it has been soon recognized NSPT in gauge field theories is hopeless without introducing any kind of stochastic gauge fixing (see below). In fact, even if this is not necessary in principle, diverging fluctuation will soon dominate without it [4,5]. A natural way of doing it is to apply (after every Langevin evolution step) a gauge trasformation $U_\mu^w(n) = e^{w(n)} U_\mu(n) e^{-w(n+\hat{\mu})}$ where $w(n) = -\frac{\alpha}{2} \sum_\mu \partial_\mu^L (U_\mu(n) - U_\mu^\dagger(n))_{\rm traceless}$ and $(0 < \alpha < 1)$. It is easy to verify that such a trasformation attracts the system towards configurations where the norms of the fields are under control. One can also apply a slightly modified version of the Landau gauge prescription (because of the presence of a non trivial vacuum), however this produces no significant difference in the results.

Unfortunately the results for the observable defining $\bar{g}^2(L)$ are extremely noisy, preventing any Symanzyk analysis to get a continuum limit. Perturbative calculations had been carried on analytically in [6] with very high precision to the second non trivial order. These known results are reproduced, for both $SU(2)$ and $SU(3)$, and it would be easy to go to higher orders. However the errorbars makes these results useless in the context they are interesting.

Two considerations are in order: first the imprecision is due essentially to the observable and not to the new vacuum configuration upon which perturbations are made, nor to the different choice of gauge fixing one is forced to take. In fact simulations of the plaquette have the same precision we were used to. This suggests that this approach could be more profitably applied to observables build up with Wilson loop, leaving still the freedom of choice of other conditions. Secondly we convinced ourself that a great help could come from a more efficient procedure of gauge fixing [7]. This is the subject of what follow.

5. Fadeev & Popov in NSPT

We now propose a new formalism which is even farer from the original spirit [8] ("Perturbation Theory without Gauge Fixing"), but implements the Fadeev & Popov mechanism in a (perturbative) Monte Carlo. Such a formalism consists in the application to NSPT of a technique which is well known in the standard Langevin equation [9]

and is in principle a solution for NSPT "dynamical fermions" as well.

Consider any problem of the class $e^{-S}\det M$ (fermions, F& P). In principle $e^{-S}\det M \sim e^{-(S-Tr\log M)}$. A solution in the standard Langevin equation has been known for a long time:consider $U(t+1) = e^{-fT}U(t)$ where $f_i = \epsilon[\partial_i S_g - \Re[\xi_k^\dagger M_{kl}^{-1}\partial_i M_{ln}\xi_n]]$ and ξ_k (k multi index) are Gaussian as well with $\langle|\xi|^2\rangle = 1$. Again to $O(\epsilon)$ what matters in Fokker–Plank equation is (average on ξ) $\langle f_i\rangle = \epsilon\partial_i[S_g - Tr\log M] + \sqrt{\epsilon}\eta$ (where η are Gaussian as well with $\langle\eta^2\rangle = 2$) which means for the probability density of the fields $P \sim e^{-(S-Tr\log M)} = e^{-S}\det M$

Instead of the fermion determinant consider the functional integral which is relevant for PT in covariant gauges (i.e. Fadeev & Popov mechanism) $\langle O\rangle = 1/Z\int DU e^{-(S_g+S_{gf})}\Delta_{FP}[U]O[U]$ where S_g = gauge Wilson action, S_{gf} is the gauge fixing action, and $\Delta_{FP}[U]$ the Fadeev Popov determinant. S_{gf} is in fact the action in charge of "killing the divergences" in Langevin equation.

It is necessary to compute the contributions to the equation of motion coming both from S_{gf} and from Δ_{FP} The first gives no problem. As far as the second is concerned notice that of course nobody implements Δ_{FP} via $\Delta_{FP} = \det L = e^{Tr\log L}$. The resulting action would be strongly non local. Instead $\det L$ is in standard applications obtained by a functional integral over grassmann fields (ghosts). Shadow of the non locality of the would be $S_{FP}[U]$ is L^{-1} in $f_i = \epsilon[\partial_i(S_g+S_{gf}) - \Re[\xi^\dagger L^{-1}\partial_i L\xi]]$ but the crucial observation is now that $L^{-1} = L_0^{-1} - gL_0^{-1}L_1L_0^{-1} + g^2(L_0^{-1}L_1L_0^{-1}L_1L_0^{-1} - L_0^{-1}L_2L_0^{-1}) + O(g^3)$. The message is that L^{-1} has a simple recursive form easy to guess also at this order; only L_0^{-1} is a proper inverse, which is well known and easy in Fourier space and it does not depend on the fields.

6. Conclusions

NSPT proved successful in performing high loop computation in lattice QCD. The precision demostrated in the computation of different quantities show that the task of pushing one loop further the evaluation of $SU(3)$ ($SU(2)$) β function is attainable.

However the choice of the scheme based on the Schroedinger Functional (which has many desirable features) proved unlucky for our approach. The direction to pursue is now double. On one side one can examine different schemes of renormalization (for instance based on Wilson loops), where NSPT calculations are more precise, and determine in this way the next coefficient of the β function. On the other side one can look at the possibility of implementing the Fadeev & Popov gauge fixing in NSPT. One notes that contributions to L^{-1} are out of a recursive structure and depend only on one inverse L_0^{-1}, which is easy to manage in Fourier space. L_0^{-1} does not depend on the fields. Because of the former observation, the actual implementation of the proposed method depends strongly on the architecture of the computer: without a good FFT it is hopeless. Same sort of remarks apply to including dynamical fermions contributions as well.

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